

Citation for Yitang Zhang

The Ostrowski Prize for 2013 is awarded to Professor Yitang Zhang for his breakthrough work on small gaps between prime numbers. Yitang Zhang was granted a Bachelor's degree in 1982 and a Master's degree in 1985 from Peking University. He then travelled to the United States for further graduate work and in 1991 obtained a Ph.D. degree from Purdue University. In 1999 he joined the Department of Mathematics at the University of New Hampshire, where he remains to this day.

Professor Zhang's work deals with a fundamental question about the distribution of prime numbers. Let us put it in a historical context. Let p_1, p_2, \dots denote the increasing sequence of prime numbers. It follows from the prime number theorem that the average gap between consecutive prime numbers p_{n+1} and p_n is roughly $\log p_n$ in size. What can be said about small gaps between consecutive primes? Erdős, in 1940, was the first to prove that there is a positive number c which is smaller than 1 such that

$$p_{n+1} - p_n < c \log p_n \tag{1}$$

for infinitely many positive integers n . This result was refined by Bombieri and Davenport, Huxley, Maier and others. In particular, Maier proved that (1) holds with $c = .248\dots$ in 1988. Next Goldston, Pintz and Yildirim established, in a pair of papers which appeared in 2009 and 2010, that a much stronger result is true. They proved that there is a positive number C such that

$$p_{n+1} - p_n < C(\log p_n)^{1/2}(\log \log p_n)^2$$

for infinitely many positive integers n . Building on the work of Goldston, Pintz and Yildirim, Zhang proved in 2013 that

$$p_{n+1} - p_n < 7 \cdot 10^7$$

for infinitely many positive integers n . This represents a stunning step forward and brings the twin prime conjecture into view. Zhang's proof employs many powerful ideas from analytic number theory including the sieve of Goldston, Pintz and Yildirim, the Bombieri-Vinogradov Theorem, Weil's bound for Kloosterman sums, Deligne's proof of the Riemann Hypothesis for varieties over finite fields and the work of Bombieri, Friedlander and Iwaniec on the distribution of primes in arithmetical progressions. It is a landmark achievement.