Ostrowski Award for 1992

The Ostrowski Award for 1992 was given to Jean Bourgain for his contributions to various areas of mathematical analysis: harmonic analysis, ergodic theory, complex analysis, functional analysis and classical convexity theory.

In all these areas J. Bourgain solved dozens of central long standing open problems and developed new powerful research tools and directions. In his work he uncovered numerous new and surprising connections between these various subjects. As a result of Bourgain's achievements several large domains in these areas of analysis changed completely and landed on a much higher plateau than before.

Among the specific results of Bourgain the following four were singled out.

1. A breakthrough in ergodic theory was Bourgain's results on subsequences. Let T be a measure preserving transformation on a probability space Ω. Then for every $f \in L_p(\Omega), 1 < p$, the sequence

$$A_N f(x) = N^{-1} \sum_{n=1}^{N} f(T^{n^2} x) \qquad , x \in \Omega$$

converges almost everywhere as $N \to \infty$. This result, solving problems of Erdös, Fürstenberg and others was proved by using tools from harmonic analysis, analytical number theory and of course ergodic theory.

The proof works in a more general situation where the sequence n^2 in the definition of A_N is replaced by e.g. $p(n)$ where p is a general polynomial with integer coefficients or the n'th prime number. Also the result generalizes to a setting where T is replaced by a finite family of commuting transformations. Bourgain's main paper on this result is: "Pointwise ergodic theorems for arithmetic sets", Pub. Math. I.H.E.S. 69 (1989) 5–45.

2. Let G be a compact commutative abelian group (e.g. the circle). Let Γ be the character group of G and let $\infty > p > 2$. A subset A of Γ is called a $\Lambda(p)$ set if there exists a constant C so that

$$\| \sum_{\gamma \in A} \alpha_\gamma \gamma(x) \|_p \le C (\sum_{\gamma \in A} |\alpha_\gamma|^2)^{1/2}$$

for every choice of scalars $\{\alpha_\gamma\}_{\gamma\in A}$. Combinatorial methods were known for construction of "large" $\Lambda(p)$ sets for p an even integer. It was a famous open problem in harmonic analysis (going back at least to Rudin's papers in 1960) whether there is e.g. a $\Lambda(3)$ set which is not already a $\Lambda(4)$ set. By using hard analytic and deep probabilistic arguments, Bourgain solved this problem. He produced for every infinite group G, every $p > 2$, and every $\varepsilon > 0$ a subset of the dual of G which is a $\Lambda(p)$ set but not a $\Lambda(p+\varepsilon)$ set. His proof works in the setting of general orthonormal series and this led him to settle several well known open problems of the Russian school on orthogonal series. Bourgain's main paper on this subject is: "Bounded orthogonal systems and the Λ_p - set problem", Acta Math. (Djursholm) 162 (1989), 227–245.

3. The following was a well known open problem in harmonic analysis in R^2 related to differentiability theory. Let f be a continuous function on the plane. Let F be the following "maximal" function. $F(x)$ is the maximum of the average values of f with respect to circles centered at x (the maximum is of course with respect to the radius). Is there a way to estimate the "size" of $F(x)$ by that of f? In dimensions $n > 2$ it was known that the answer is positive since L_2 estimates were available. It was also known that in R^2 the easy to handle L_2 estimates fail. Bourgain solved this problem by proving L_p estimates for $p < 2$ using a very delicate geometric argument (which naturally works also for more complicated situations than that of the circle). His result solved (negatively) in particular the following problem which was well known in the theory of fractal sets. Does there exist a set of circles in R^2 so that the union of the circles has measure 0 while the set of the centers has positive measure? Bourgain's main paper on this subject is: "Averages in the plane over convex curves and maximal operators", J. d'Analyse 47(1986), 69–85.

4. In most surveys or problem sets in harmonic analysis a prominent role is played by problems on oscillatory integrals and in particular high dimensional Bochner Riesz multipliers due to Hörmander, Fefferman, Stein and others (see e.g. Stein's paper: Some problems in harmonic analysis, Proc. Symposia A.M.S. vol 35, 1979). The only known positive results for $R^n, n > 2$, were L_2 estimates and what follows from them by interpolation. In a major breakthrough Bourgain obtained recently results which went beyond L_2 estimates in a paper entitled "L^p - estimates for oscillatory integrals in several variables"

Geom. and Funct. Anal. 1 (1991), 321–374. This paper contains the first new definitive result in this direction (which is of importance also in P.D.E.) in 20 years. It gives the best possible result for a certain range of p's (depending on the dimension) though not for all p. The proof depends among other things on new structural results on Kakeya sets in R^n for $n \geq 3$.